Hypersonic Vehicle Control Law Development Using H_∞ and μ-Synthesis

Irene M. Gregory, John D. McMinn and John D. Shaughnessy

NASA Langley Research Center

and

Rajiv S. Chowdhry

Lockheed Engineering Sciences, Co.

**NASA Langley GN&C Workshop
March 18-19, 1993**

Hypersonic Vehicle Control Law Development Using H_∞ and μ-Synthesis

Irene M. Gregory, John D. McMinn and John D. Shaughnessy
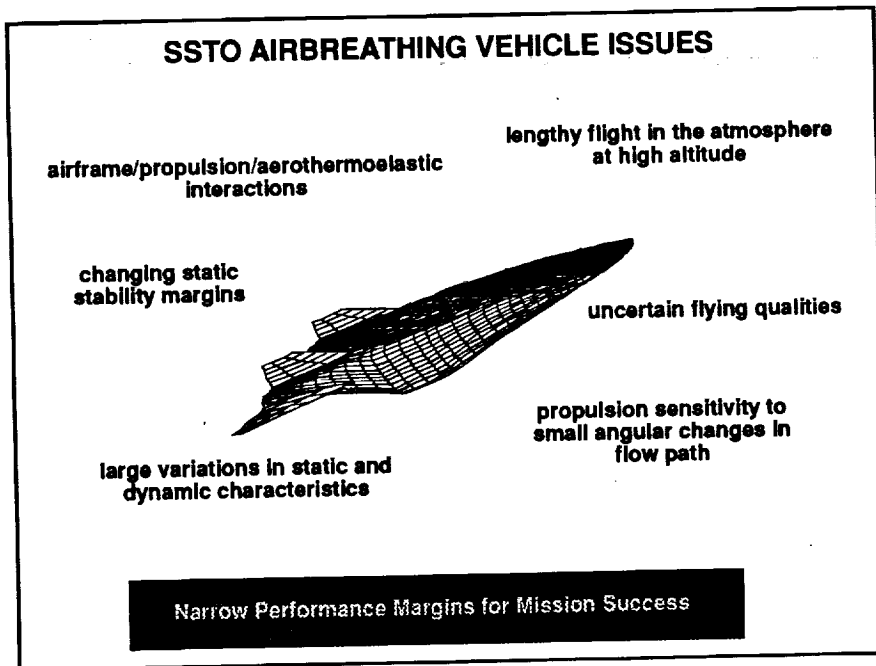
NASA Langley Research Center

and

Rajiv S. Chowdhry

Lockheed Engineering Sciences, Co.

Airbreathing SSTO vehicle has a multi facetted mission that includes orbital operations, as well as re-entry and descent culminating in horizontal landing. However, the most challenging part of the operations is the ascent to orbit. The airbreathing propulsion requires lengthy atmospheric flight that may last as long as 30 minutes and take the vehicle half way around the globe.

The vehicle's ascent is characterized by tight payload to orbit margins which translate into minimum fuel to orbit as the performance criteria.

SSTO AIRBREATHING VEHICLE ISSUES

airframe/propulsion/aerothermoelastic interactions

lengthy flight in the atmosphere at high altitude

changing static stability margins

uncertain flying qualities

large variations in static and dynamic characteristics

propulsion sensitivity to small angular changes in flow path

Narrow Performance Margins for Mission Success

The lengthy atmospheric flight and the minimum fuel to orbit performance requirement lead to a number of issues. Among these issues are:

• Large variations in static and dynamic vehicle characteristics that result from large and rapid mass change as well as aerodynamic heating.

• These variations lead to changing static stability margins as the aerodynamic center of pressure moves significantly with respect to c.g..

• Furthermore, since the undersurface of the vehicle serves as the compressing inlet and as the nozzle for the propulsion system, this vehicle experiences unprecedented degree of airframe/propulsion/aerothermoelastic interactions that lead to multiple and large parametric uncertainty.

• The lengthy atmospheric flight subjects the vehicle and the propulsion system to atmospheric turbulence which can excite vehicle dynamic modes as well as degrade propulsion performance through large density variations.

• Propulsion system itself is sensitive to small angular changes in the flow path that may be caused by interactions or atmospheric turbulence.

• Finally, flying qualities for hypersonic flight are not yet established.

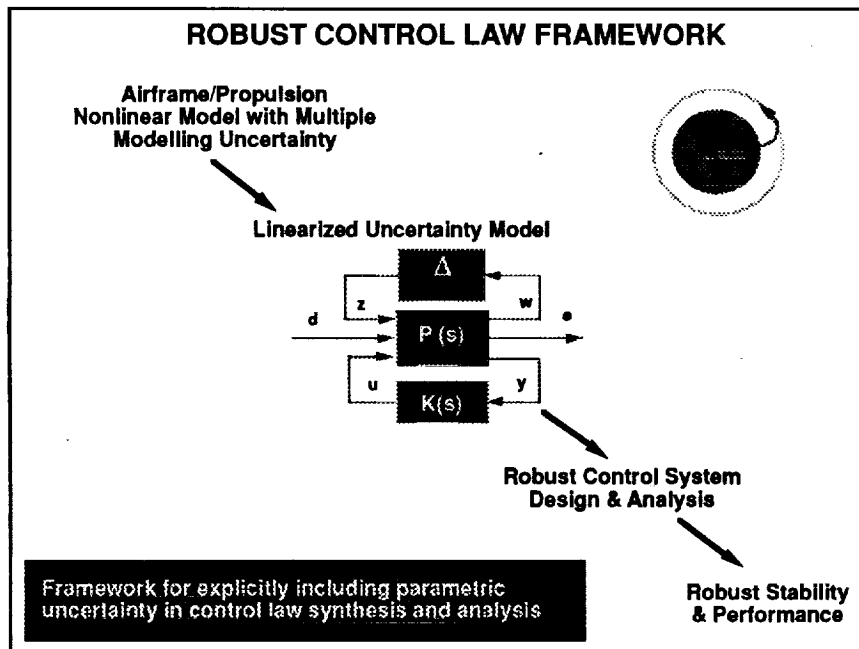All these issues lead to narrow margins for mission success making optimal vehicle performance absolutely essential.

**CONTROL SYSTEM PERFORMANCE
REQUIREMENTS**

- Tight Performance Margins For Reaching Orbital Speeds

- Closely Tracking Optimal Trajectory

- Tight Angle Of Attack Envelope - 0.5 Deg

- Atmospheric Disturbance Rejection

- Robustness To Parameter Uncertainty

In order to address these issues and enhance vehicle performance, the control system must satisfy the following requirements. It must :

- stabilize the vehicle,

- precisely track optimal fuel trajectory,

- attenuate atmospheric disturbances,

- while minimizing control effort since even moderate elevon deflections result in very large integrated drag penalty.

- All of these performance requirements must be satisfied in the presence of parametric uncertainty

And, as with all piloted vehicles, the flying qualities requirements must be met.
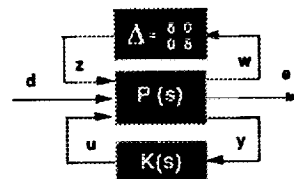
ROBUST CONTROL LAW FRAMEWORK

Airframe/Propulsion Nonlinear Model with Multiple Modelling Uncertainty

Linearized Uncertainty Model

Δ

z   w

d

P (s)

u   y

K(s)

Robust Control System Design & Analysis

Framework for explicitly including parametric uncertainty in control law synthesis and analysis

Robust Stability & Performance

Recent advances in H∞/μ robust control theory provide a framework for explicitly including parametric uncertainty in control law design. The hypersonic vehicle ascent can be characterized in this framework as follows.

• We begin with aero/propulsion nonlinear model that includes multiple sources of uncertainty.

• This model is translated into a linearized uncertainty model illustrated by the diagram in the middle of the slide. The linear model itself is contained in plant P. All the uncertainty, with the physical relationship to the model preserved, is collected in block Δ. The controller K is then designed and analyzed for this linearized uncertainty model.

Once this process is successfully completed, the resulting control law provides robust stability and desired performance in the presence of specified uncertainty.

APPLICATION EXAMPLE

- **Flight condition**
  - 2000 psf $\bar{q}$ trajectory
  - Mach 8

- **Plant - P (s)**
  - linear model of the vehicle
  - design specifications

- **Uncertainty block - Δ**
  - uncertainty in control effectiveness of elevon and fuel flow rate

- **Exogenous inputs - d**
  - commanded variables - velocity and altitude
  - atmospheric turbulence
  - sensor noise

- **Performance outputs - e**
  - velocity and altitude error
  - angle of attack
  - control effector - elevon and fuel flow rate

To explore this robust controls framework further, an example representing a number of issues we discussed has been selected. The vehicle is a conical configuration following a 2000 psf dynamic pressure trajectory and accelerating through Mach 8 at the design point.

We return to the Linearized Uncertainty Model diagram from the previous slide to illustrate how this example fits into the framework.

The plant P contains the linear model of the vehicle and the design specifications, which will be discussed in more detail later. All the uncertainty in the problem is relegated to the effectiveness of elevon and fuel flow rate, and is contained in the uncertainty block Δ. The physical inputs, d, into the system include the commanded variables, velocity and altitude, atmospheric turbulence, and sensor noise. The performance outputs, e, are velocity and altitude error, angle of attack, due to propulsion performance sensitivity to this quantity, and control effectors, both deflection and rate for elevon and fuel flow rate.

```
┌─────────────────────────────────────────────────────────────┐
│                    PROBLEM FORMULATION                        │
│                                                               │
│  · Applicability and effectiveness of H∞/μ controller         │
│     - Performance                                             │
│     - Stability                                               │
│     - Changing flight conditions                              │
│     - Changing vehicle characteristics                        │
│                                                               │
│  · Performance metric                                         │
│     - Velocity and altitude                                   │
│        · 5 % steady state error                               │
│        · 10 % overshoot                                       │
│        · 40 sec time constant                                 │
│     - Angle of attack                                         │
│        · limit to 0.5 deg peak-to-peak deflection             │
│     - Elevon                                                  │
│        · limit deflections to 2 deg                           │
│                                                               │
│   H∞/μ controller designed for worst possible combination of inputs │
└─────────────────────────────────────────────────────────────┘
```

We are interested in establishing how effective is each technique, that is H∞ and μ, in explicitly dealing with changing vehicle characteristics and flight conditions while providing performance and stability.

Performance on the global level refers to achieving minimum fuel to orbit. On the more immediate design level it encompasses a metric such as illustrated on this slide. The time domain response specifications, limits on the deflection and rate of the control effectors, and atmospheric turbulence attenuation are all serve as performance specifications.

It is important to point out that unlike other optimal robust control methods, H∞ based design results in a controller for the worse case input combination.
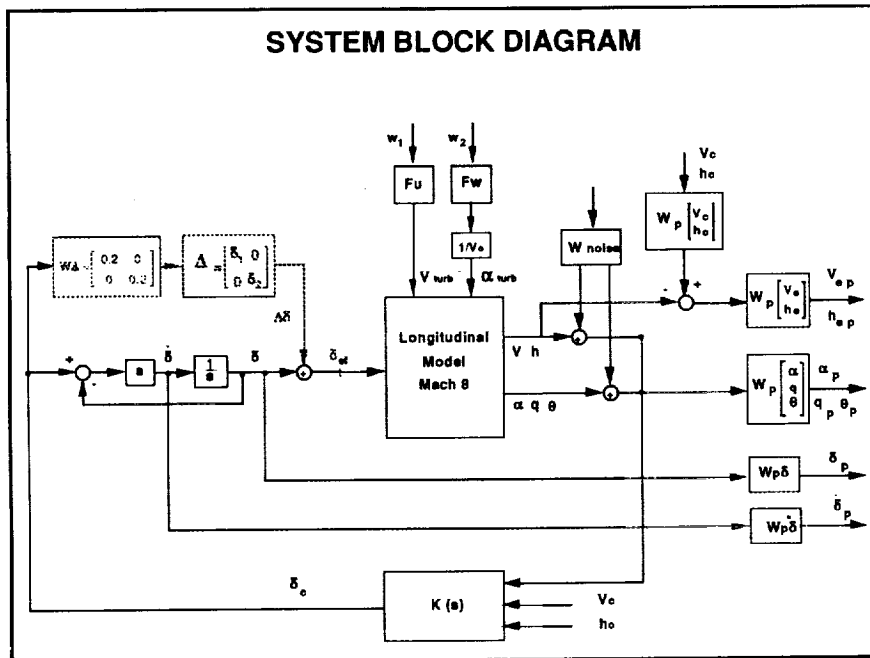
DESIGN SPECIFICATIONS

- Time domain specs translated into frequency domain
  - steady state error, % overshoot, time constant --> transfer function

- Disturbance attenuation --> low frequency transfer function gain

- Rate and position limits on actuators --> transfer function

- Allowable uncertainty
  - % parameter variation
  - frequency dependent transfer function

Design goals handled with frequency domain weightings

I like to spend a moment discussing how we fit design specifications discussed few slides back into the H∞ context. To fully exploit H∞ capabilities, design specifications must reflect the desired performance as closely as possible. Given the specifications on the time domain response, steady state error, percent overshoot, and time constant, translate directly into a transfer function that is utilized in H∞ context. The same can be said about performance specifications on alleviating atmospheric turbulence and limiting rate and position of actuators.

The allowable uncertainty in the system is also specified in frequency domain as a percent of nominal. It is either a constant across all frequency or varies depending on type of uncertainty.
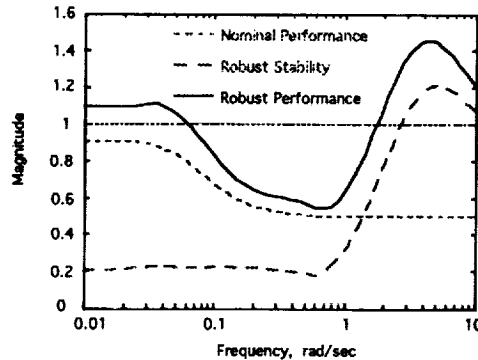
At this juncture, we would like to examine how all this relates to a standard block diagram.

SYSTEM BLOCK DIAGRAM

The three block general structure that you may recall from previous slides is expanded into this block diagram for our example. The dashed boxes, primarily on the right side of the diagram, represent the performance specifications. The dotted boxes, on the left, represent the uncertainty. The 20 percent uncertainty that was just discussed is expressed by the matrix WΔ and the diagonal structure of Δ reflects that each actuator effectiveness is independent of the other.

Now with problem formulated we design a controller which is analyzed in the following slides.

H∞ CONTROLLER FREQUENCY ANALYSIS

- **Nominal Performance**
  - performance of the nominal plant model
- **Robust Stability**
  - stability in the presence of allowable uncertainty
- **Robust Performance**
  - performance in the presence of allowable uncertainty

The first to be analyzed is the H∞ controller. Very briefly to provide you with a point of reference. 1 delineates the boundary between successfully passing a given test vs. failing it. We are interested in three metrics for this controller. The first is nominal performance which tells us whether the desired performance has been achieved under ideal conditions, in other words, we have no uncertainty in our system. As you can see from this plot, nominal performance is less than 1, therefore satisfying our desired performance requirements.

But since no realistic system model is ideal, we are really interested in its behavior in the presence of uncertainty. For this example it constitutes 20 percent control effectiveness uncertainty. The initial interest is in stability. This controller violates robust stability criteria around 4 rad/sec. As expected, the level of desired performance in the presence of this uncertainty is also not achieved.

At this point, we have two options - to relax the uncertainty and performance specifications or to see if μ controller can provide the desired robust performance.
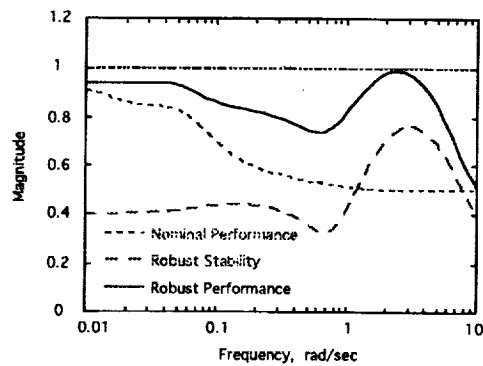
μ CONTROLLER FREQUENCY ANALYSIS

Control system satisifes performance requirements in the presence of 20 % control effectiveness uncertainty

The μ controller designed to handle 20 percent control effector uncertainty satisfies all three metrics. The level of desired performance in the presence of specified uncertainty is achieved with some margin to spare. We would like to see how much uncertainty can be tolerated and still satisfy robust performance.
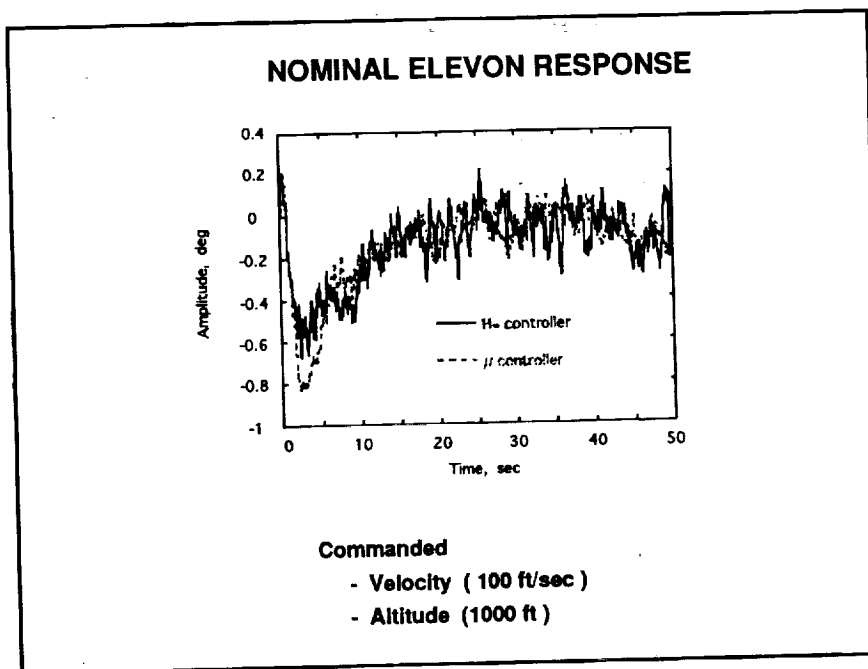
μ CONTROLLER FREQUENCY ANALYSIS

Control system satisfies performance requirements upto
40 % uncertainty in the control effectiveness

The maximum level is achieved at 40 percent uncertainty. In fact, this μ controller satisfied robust performance for up to 40 percent uncertainty in control effectiveness as indicated in this plot.

No analysis is complete without looking at the actual time histories. So to validate and to augment conclusions from frequency analysis, a sample of time responses is presented.

NOMINAL ELEVON RESPONSE

Commanded
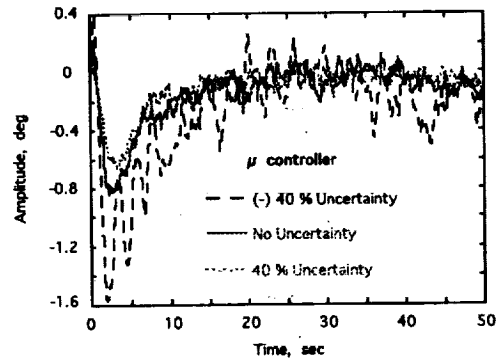- **Velocity (100 ft/sec)**
- **Altitude (1000 ft)**

I like to point out that in all time responses you will see, the vehicle is commanded to simultaneously increase velocity and altitude while being subjected to moderate atmospheric turbulence.

The first plot is elevon response for both H∞ and μ controllers for an ideal system, i.e. no uncertainty present. Note that for both, initial deflection is less than one degree. Important fact is that both responses are very similar, indicating that improved robustness is achieved at a small loss in ideal performance as measured by the total deflection.

Introducing 20 percent uncertainty into the system drives H∞ controller unstable, which leaves us with μ controller response to consider.

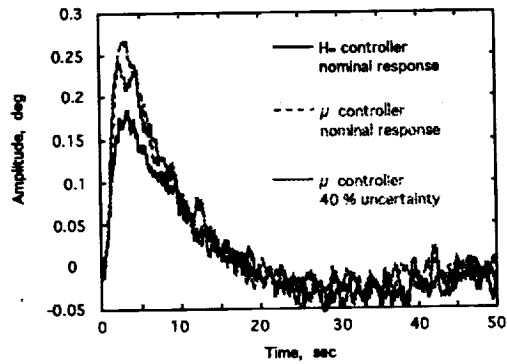## WORST CASE PERFORMANCE ELEVON RESPONSE

Commanded
- Velocity (100 ft/sec)
- Altitude (1000 ft)

Elevon and Fuel Flow Rate Uncertainty

If we look at worst case while the desired performance is still achieved, we get this plot. Recall that our performance specifications were still satisfied for 40 percent uncertainty. The amplitude of the response is dependent on the positive or negative uncertainty in the control effectiveness. In the worse case scenario when the actual effectiveness is 40 percent less than ideal, the elevon deflection is still less than 2 degrees which was the limit.

Well how does this behavior impact other performance variables of interest.

ALPHA NOMINAL AND WORST CASE RESPONSE

Commanded
- **Velocity (100 ft/sec)**
- **Altitude (1000 ft)**

The commanded change in altitude which is facilitated by elevon deflection is unaffected by the uncertainty. In fact, it is faster and more precise for the μ controller than for the ideal H∞ one. But this performance improvement in altitude does have an adverse effect on another variable of importance - angle of attack.

The μ controller angle of attack peak is somewhat higher than that of the H∞ controller, though both responses are well within the specified limit of 0.5 degree. The uncertainty again has very small effect on the μ controller response

CONCLUSIONS

- Vehicle characteristics and control system requirements translate explicitly into H_∞ domain specifications

- H_∞ controller suffers performance degradation with introduction of control effectiveness uncertainty

- μ-synthesis results in an improved robust performance over H_∞ controller

μ Analysis And Synthesis Technique Provides A Systematic Approach To Explore Tradeoff Between Performance And Uncertainty Robustness

So what have we learned from this initial application of H∞ and μ to an airbreathing SSTO vehicle. The bottom line is that μ framework provides a systematic approach to include parametric uncertainty in design and to explore tradeoffs between performance and uncertainty robustness. This initial application of μ synthesis and analysis techniques to an airbreathing SSTO shows much promise.